

## PROBLEMS FOR ALGEBRA SECTION

### 1. INDIVIDUAL TEST

**Problem 1** (Individual 1). Let  $K$  be the splitting field of  $f(x) = x^4 - 4x^2 - 1$  over  $\mathbb{Q}$ .

- (1) Show that  $f(x)$  is irreducible over  $\mathbb{Q}$ .
- (2) Describe the Galois group  $\text{Gal}(K/\mathbb{Q})$ .

**Problem 2** (Individual 2). Let  $\overline{\mathbb{F}}_p$  be an algebraic closure of  $\mathbb{F}_p$  ( $p$  is a prime). Describe the abelian group  $\overline{\mathbb{F}}_p^\times$  in more elementary terms. What is the action of the Frobenius in terms of your description?

**Problem 3** (Individual 3). Let  $A$  and  $B$  be two  $n \times n$  matrices with coefficients in  $\mathbb{Q}$ . For any field extension  $K$  of  $\mathbb{Q}$ , we say that  $A$  and  $B$  are similar over  $K$  if  $A = PBP^{-1}$  for some  $n \times n$  invertible matrix  $P$  with coefficients in  $K$ . Prove that  $A$  and  $B$  are similar over  $\mathbb{Q}$  if and only if they are similar over  $\mathbb{C}$ .